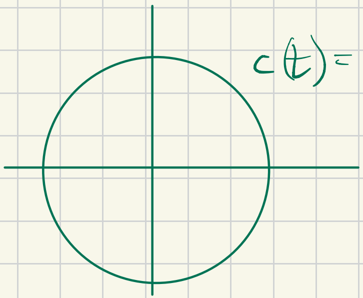
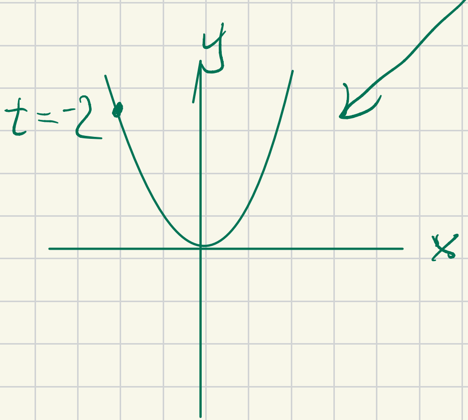
2.4 Paths and curves

We all know what curves are. Often we can describe them in more than one way:

- as points satisfying an equation, like y = x^2
- parametrically, like c(t) = (t,t^2)



$c(t) = (\cos t, \sin t)$



This section is about the parametric description. It provides a different way to describe functions c : R -> R^n, and also a way to understand the curves.

Terminology: a path is a mapping
c : [a,b] -> R^n $\quad [a, b] \subseteq \mathbb{R}$

The velocity vector of a path $c(t)$.

If $c(t) = (c_1(t), \ldots, c_n(t))$ are the component functions, the velocity of c is
$c'(t) = (c_1'(t), \ldots, c_n'(t))$

It's length is the speed. It points in the tangent direction to the curve at $c(t)$.

Typical question: Consider the path
$c(t) = t(\cos(t), \sin(t))$

a. Find the velocity vector of c at $t=2$

$$c'(t) = (\cos t - t\sin t, \ \sin t + t\cos t)$$

b. Find the equation of the tangent line to the curve at $c(2)$.

$$v = c(2) + t c'(t)$$